THE STORY OF ECONOPHYSICS

BIKAS K. CHAKRABARTI*,†, AND ARNAB CHATTERJEE‡

Introduction

The entry on "Econophysics" by economist J. Berkley Rosser, Jr. in the New Palgrave Dictionary of Economics[1] begins with the statement "According to Bikas Chakrabarti (...), the term 'econophysics' was neologized in 1995 at the second Statphys-Kolkata conference in Kolkata (formerly Calcutta), India, by the physicist H. Eugene Stanley ...". The econophysics research therefore formally completes fifteen years of research by the end of this year! Indeed it is gratifying to see that the revered physics journal Reviews of Modern Physics has published already their first review on econophysics, namely on Statistical mechanics of money, wealth, and income by physicist Victor Yakovenko and economist J. Barkley Rosser, Jr.[2] late last year!

We intend to trace these developments. In the following three sections we first give a brief history of economics (from outsider's point of view; essentially following ref.[3]), compare next the developments in econophysics with those in biophysics and geophysics (again following ref.[4]) and then present briefly the story of Econophysics in Kolkata (essentially following ref.[5]). We finally discuss the econophysical challenges of "New Economic Thinking"[6] in the context of the development of the "Institute for New Economic Thinking"[7].

* Theoretical Condensed Matter Physics Division and Centre for Applied Mathematics and Computational Science, Saha Institute of Nuclear Physics, 1/AF, Bidhannagar, Kolkata 700 064, India. Email: bikask.chakrabarti@saha.ac.in

† Economic Research Unit, Indian Statistical Institute, 203 Barrackpore Trunk Road, Kolkata 700 108, India.

‡ CMSP Section, The Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, Trieste I-34014, Italy. Email: achatter@ictp.it

A Brief History of Economics: Outsiders' Account

When physics started to develop, during the time of Galileo Galelei (1564-1642), there was hardly any science at a grown-up stage to get help or inspiration from. The only science that was somewhat mature was mathematics, which is an analytical science (based on logic) and not synthetic (based on observations/ experiments carried out in controlled environments or laboratories). Yet, developments in mathematics, astronomical studies in particular, had a deep impact in the development of physics, of which the (classical) foundation was almost single-handedly laid down by Isaac Newton (1643-1727). Mathematics remained at the core of physics since then. The rest of main stream sciences, like chemistry, biology etc all tried to get inspiration from, utilize, and compare with physics since then.

In principle, development in social sciences started much later. Even the earliest attempt to model an agricultural economy in a kingdom, the "physiocrats' model", named after the profession of its pioneer, the French royal physician Francois Quesnay (1694-1774), came in the third quarter of the eighteenth century when physics was already put on firm ground by Newton. The physiocrats made the observation that an economy consists of obvious components like land and farmers. Additionally, they identified the other components as investment (in the form of seeds from previous savings) and protection (during harvest and collection, by the landlord or the king). The impact of the physical sciences, in emphasizing these observations regarding components of an economy, is clear. Analogy with human physiology suggested that, like the healthy function of a body required proper functioning of each of its components or organs and the (blood) flow among them remaining uninterrupted, each component of the economy should be given proper care (suggesting rent for land and tax for protection!). Although the physiocrats' observations were appreciated later, the attempt to conclude using the analogy with human physiology was not.

Soon, at their last phase, Mercantilists, like Wilhelm von Hornick (1638-1712), James Stewart (1712-1780) et al, made some of the most profound and emphatic observations in economics, leading to the foundation of political economy. In particular, the observations by the British merchants (who traded in the colonies, including India, in their own set terms) that instability/unemployment growing at their home country in years whenever there had been a net trade deficit and out-flow of gold (export being less than import). This led to the formulation of the problem of effective demand: even though the merchants, or traders were independently trading (exporting or importing goods) with success, the country's economy as a whole did not do well due to lack of overall demand when there was a net flow of gold (the international exchange medium) to balance the trade deficit! This remains still a major problem in macroeconomics. The only solution in those days was to introduce tax on import: the third party (namely the government) intervention on individuals' choice of economic activity (trade). This immediately justified the involvement of the government in the economic activities of the individuals.

In a somewhat isolated but powerful observation, Thomas Malthus (1766-1834) made a very precise modeling of the conflict between agricultural production and population growth. He assumed that the agricultural production can only grow (linearly) with the area of the cultivated land. With time t, say year, the area can only grow linearly ($\alpha\ t$) or in arithmetic progression (AP). The consumption depends on the population which, on the other hand, grows exponentially ($\exp[t]$) or in geometric progression (GP). Hence, with time, or year 1, 2, 3, . . ., the agricultural production grows as 1, 2, 3, . . ., while the consumption demand or population grows in a series like 2, 4, 8, ... No matter, how much large area of cultivable land we start with, the population GP series soon takes over the food production AP series and the population faces a disaster — to be settled with famine, war or revolution! They are inevitable, as an exponentially growing function will always win over a linearly growing function and such disasters will appear almost periodically in time!

Adam Smith (1723-1790) made the first attempt to formulate the economic science. He painstakingly argued that a truly many-body system of selfish agents, each having no idea of benevolence or charity towards its fellow neighbors, or having no foresight (views very local in space and time), can indeed reach an equilibrium where the economy as a whole is most efficient; leading to the best acceptable price for each commodity. This 'invisible hand' mechanism of the market to evolve towards the 'most efficient' (beneficial to all participating agents) predates by ages the demonstration of 'self-organization' mechanism in physics or chemistry of many-body systems, where each constituent cell or automata follows very local (in space and time) dynamical rules and yet the collective system evolves towards a globally 'organized' pattern (cf. Ilya Prigogine (1917-2003), Per Bak (1947-2002) et al). This idea of 'self-organizing or self-correcting economy' by Smith of course contradicted the prescription of the Mercantilists regarding government intervention in the economic activities of the individuals, and argued tampering by any external agency to be counterproductive.

Soon, the problem of price or value of any commodity in the market became a central problem. Following David Ricardo's (1772-1823) formulation of rent and labour theory of value, where the price depends only on the amount of labour put by the farmers or labourers, Karl Marx (1818-1883) formulated and forwarded emphatically the surplus labour theory of value or wealth in any economy. However, none of them could solve the price paradox: why diamond is costly, while coal is cheap? The amount of labour in mining etc are more or less the same for both. Yet, the prices are different by astronomical factors! This clearly demonstrates the failure of the labour theory of value. The alternative forwarded was the utility theory of price: the more the utility of a commodity, the more will be its price. But then, how come a bottle of water costs less than a bottle of wine? Water is life and certainly has more utility! The solution identified was marginal utility. According to marginal utility theory, not the utility but rather its derivative with respect to the quantity determines the price: water is cheaper as its marginal utility at the present level of its availability is less than that for wine — will surely change in a desert. This still does not solve the problem completely. Of course increasing marginal utility creates increasing demand for it, but its price must depend on its supply (and will be determined by equating the demand with the supply)! If the offered (hypothetical) price p of a commodity increases, the supply will increase and the demand for that commodity will decrease. The price, for which supply S will be equal to demand D, will be the market price of the commodity: $S(p) = D(p)$ at the market (clearing) price. However, there are problems still. Which demand should be equated to which supply? It is not uncommon to see often (in India) that price as well as the demand for rice (say) increases simultaneously. This can occur when the price of the other staple alternative (wheat) increases even more.

The solutions to these problems led ultimately to the formal development of economic science in the early twentieth century by Léon Walras (1834-1910), Alfred Marshal (1842-1924) and others: marginal utility theory of price and cooperative or coupled (in all commodities) demand and supply equations. These formulations went back to the self-organizing picture of any market, as suggested by Adam Smith, and incorporated this marginal utility concept, and utilized these coupled demand-supply equations: $D_i(p_1, p_2, \ldots, p_i, \ldots, p_N, M) = S_i(p_1, p_2, \ldots, p_i, \ldots, p_N, M)$ for N commodities and total money M in the market, each having relative price tags p_i (determined by marginal utility rankings) and demand D_i and supply S_i; $i = 1, 2, \ldots, N$ and the functions D or S are in general nonlinear in their arguments. These formal and abstract formulations of economic science were not appreciated very much in its early days and had a temporary setback. The lack of acceptance was due to the fact that neither utility nor marginal utility is measurable and the formal solutions of these coupled nonlinear equations in many (p_i) variables still remain elusive. The major reason for the lack of appreciation for these formal theories was a profound and intuitive observation by John Maynard Keynes (1883-1946) on the fall of aggregate (or macroeconomic) effective demand in the market (as pointed out earlier by the Mercantilists; this time due to 'liquidity preference' of money by the market participants) during the great depression of 1930's. His prescription was for government intervention (in direct contradiction with the 'laissez-faire' ideas of leaving the market to its own forces to bring back the equilibrium, as Smith, Walras et al proposed) to boost aggregate demand by fiscal measures. This prescription made immediate success in most cases. By the third quarter of the twentieth century, however, its failures became apparent and the formal developments in microeconomics took the front seat again.

Several important, but isolated observations in the meantime contributed later very significantly. Vilfredo Pareto (1848-1923) observed that the number density $P(m)$ of riches in any society decreases rather slowly with their richness m (measured in those days by palace sizes, number of horses, etc of the kings/landlords in all over Europe): $P(m) \sim m^{-\alpha}$; for very large m (very rich people); $2 < \alpha < 3$ (Cours d'Economic Politique, Lausanne, 1897). It may be mentioned, at almost the same time, JoshiahWillard Gibbs (1839-1903) had put forward precisely that the number density $P(\varepsilon)$ of particles (or microstates) with energy ε in a thermodynamic ensemble in equilibrium at temperature T falls off much faster: $P(\varepsilon) \sim \exp[-\varepsilon/T]$ (Elementary Principles of Statistical Mechanics, 1902). This was by then rigorously established in physics. The other important observation was by Louis Bachelier (1870-1946) who modeled the speculative price fluctuations (σ), over time τ, using a Gaussian statistics (for random walk): $P(\sigma) \sim \exp[-\sigma^2/\tau]$ (Thesis: Théorie de la Spéculation, Paris, 1900). This actually predated Albert Einstein's (1879-1955) random walk theory (1905) by five years. In another isolated development, mathematician John von Neumann (1903-1957) started developing the game theories for microeconomic behavior of partners in oligopolistic competitions (to take care of the strategy changes by agents, based on earlier performance).

In the mainstream economics, Paul Samuelson (1915-) investigated the dynamic stabilities of demand-supply equilibrium by formulating, following Newton's equations of motion in mechanics, dynamical equations $\frac{dD_i}{dt} = \sum_i J_{ij} D_j\,(p_1, p_2, \cdots p_N, M)$ and $\frac{dS_i}{dt} = \sum_i K_{ij}\, S_i\,(p_1, p_2, \cdots p_N, M)$, with the demand and supply (overlap) matrices J and K respectively for N commodities, and by looking for the equilibrium state(s) where $dS/dt = 0 = dD/dt$ at the market clearing prices $\{p\}$where $D_i(\{p\},M) = S_i(\{p\},M)$. Jan Tinbergen (1903-1994), a statistical physicist (student of Paul Ehrenfest of Leiden University) analyzed the business cycle statistics and initiated the formulation of econometrics. By this time, these formal developments in economics, with clear impact of other developed sciences (physics in particular), were getting recognized. In fact, Tinbergen was the first recipient of the newly instituted Nobel prize in Economics in 1969 (for other sciences, they started in 1901; a delay by 68 years in 109 years' history of the prize!) and the next year, the prize went to Samuelson. Soon, the formal developments like the axiomatic foundations of utility (ranking) theory, and solution of general equilibrium theory by Kenneth Arrow (1921-), those of George Stigler (1911-1991), who first performed Monte Carlo simulations of markets (similar to those of thermodynamic systems in physics), or that of John Nash (1928-), giving the proof of the existence of equilibrium solutions in strategic games, etc, all were appreciated by awarding the Nobel prizes in economics (in 1972, 1982 and 1994 respectively). Although the impact of developments in physics had a clear mark in those of economics so far, it was not that explicit until about a decade and a half back.

The latest developments (leading to econophysics) had of course its seed in several earlier observations. Important among them was by Benoit Mandelbrot (1924-) when he observed in 1963 that the speculative fluctuations (in the cotton market for example) have a much slower rate of decay, compared to that suggested by the Gaussian statistics of Bachelier, and falls down following a power law statistics: $P(\sigma) \sim \sigma^{-\alpha}$ with some robust exponent value (α) depending on the time scale of observations[8]. With the enormous amount of stock market data now available on the internet, Eugene Stanley, Rosario Mantegna and coworkers established firmly[9] the above mentioned (power-law) form of the stock price fluctuation statistics in late 1990's. Simultaneously, two important modeling efforts, inspired directly from physics, started: the minority game models, for taking care of contagious behavior (in contrast to perfect rational behavior) of agents in the market, and learning from the past performance of the strategies, were developed by Brian Arthur, Damien Challet, Yi-Cheng Zhang *et al*[10], starting 1994. Agent based modeling[11] proved to be quite successful in capturing the essential empirical features of markets. The other modeling effort was to capture the income or wealth distribution in society, similar to energy distributions in (ideal) gases. These models intend to capture both the initial Gamma/log-normal distribution for the income distributions of poor and middle-income groups and also the Pareto tail of the distribution for the riches. It turned out, as shown by the Kolkata group during the last half of 1990 to the first half of 2000, a random saving gas model can easily capture these features of the distribution function. However, the model had several well documented previous, somewhat incomplete, versions available for a long time. Meghnad Saha (1893-1956), the founder of Saha Institute of Nuclear Physics, Kolkata (named so after its founder's death), and collaborators, already discussed at length in their text book, in the 1950's, the possibility of using Maxwell-Boltzmann velocity distribution (a Gamma distribution) in an ideal gas to represent the income distribution in societies: "*suppose in a country, the assessing department is required to find out the average income per head of the population. They will proceed somewhat in the similar way ... (the income distribution) curve will have this shape because the number of absolute beggars is very small, and the number of millionaires is also small, while the majority of the population have average income.*" (section on 'Distribution of velocities' in A Treatise on Heat, M. N. Saha and B. N. Srivastava, Indian Press, Allahabad, 1950; pp. 132-134). This modeling had the obvious drawback that the distribution could not capture the Pareto tail. However, the accuracy of this Gibbs distribution for fitting the income data available now in the internet has been pointed out recently by Victor Yakovenko and collaborators in a series of papers since 2000. The 'savings' ingredient in the ideal-gas model, required for getting the Gamma function form of the otherwise ideal gas (Gibbs) distribution, was also discovered more than a decade earlier by John Angle. He employed a different driver in his stochastic model of inequality process. This inequality coming mainly from the stochasticity, together with the equivalent of saving introduced in the model. A proper Pareto tail of the Gamma distribution comes naturally in this class of models when the saving propensity of the agents are distributed, as noted first by the Kolkata group and analyzed by them and by the Dublin group led by Peter Richmond.

Apart from the intensive involvements of physicists together with a few economists in this new phase of development, a happy feature has been that econophysics has almost established itself as a (popular) research discipline in statistical physics. Many physics journals have started publishing papers in such an interdisciplinary field. Also, courses in econophysics are being offered in several universities, mostly in their physics departments.

In spite of all these, it must be stated that there has, so far, been no spectacular success. Indeed, the mainstream economists are yet to take note of these developments[12]. In her account, reporting on the Econophys-Kolkata I (New Scientist, UK, 12 March 2005 issue, pp.6-7), Jenny Hogan reported several criticisms by economists, mostly appreciating the observations, but not the modeling efforts! The same kind of criticism have recently been expressed more emphatically by economists Mauro Gallegati, Thomas Lux and others[13].

Econophysics and Sociophysics: Comparison with Earlier Interdisciplinary Developments

Physics, even counted from the time of Isaac Newton (1643-1727), is now about three hundred years old. It is presently the most matured, successful and dependable of all the natural sciences. It is only natural therefore that along with its own developments, physics had also explored, often very successfully, some other natural science territories and created for example the (not so unconventional any more) branches of physics like astrophysics, biophysics and geophysics. And econophysics seems to be a recent addition to this kind of endeavor.

Intense researches on astrophysics and biophysics are now conducted by the physicists in their own departments (not in astronomy or biology departments; although a few 'older' departments are still named 'Department of Physics & Astronomy' !). These research results are also published in regular physics research journals. Nobel prizes in physics have also been awarded to some of these outstanding 'interdisciplinary physics' researchers (at least four astrophysics Nobel prizes so far)! It might be noted in this connection that in astrophysics research, consistent and thorough knowledge development were appreciated from the very beginning (with appropriate emphasis on the scanty observational results available at any stage), and this also percolated in the biophysics research community recently[v] (with the 'old' hang-ups given away) and considerable progresses are being made these days (note, the change-over to the present-day molecular biology, from the cellular one, occurred through the X-ray structure determination of DNA by physicists like Francis Crick and collaborators in the mid-fifties). As such, both these research streams are now very much part of any physics departmental activity, as also of any of the professional physics journal. Although the main researches in the important area of geosciences are only physical in nature, no regular geophysicists can be commonly found in the physics departments. The same is true for geophysics research papers: they are not regularly published in standard physics journals. To our minds, excessive emphasis on and appreciation of too many disconnected observations, without any attempt to comprehend them, had been the root cause of its failure in inspiring their colleagues like physicists, or for that matter, others. And of course there has been no Nobel prize yet for geophysicists (except perhaps to Edward Appelton for ionosphere research in 1947)!!

Crudely speaking, the main-stream physics research is now composed of two major branches: one looking for the basic constituents of matter and their interactions and mechanics, and the second part deals with the collective (dynamical) properties or behavior of a 'many-body' assembly (typically of the size of Avogadro number of order 10^{23}) of such constituents. After the advent of modern computers in the last thirty years or so, considerable development in understanding these 'collective dynamics' and the consequent 'emergent features' in the dynamics of such many-body systems, especially when each of the constituent follows very simple (local in space and time) but nonlinear dynamics, has taken place. A striking observation in these studies had been the 'self-organized emergence' of 'globally tuned' patterns out of their collective dynamics and their 'universality' classes, independent of the details of the microscopic dynamics of its constituents. Understanding of the 'global' effects of the 'frustrating' constraints among the dynamics of the constituents are now reasonably matured.

All these encouraged the physicists to check and explore their earned knowledge to the well-known many-body systems in the society: like in economics and sociology. Not unlike in the previous attempts and developments, these unconventional applications also try to bring these researches (in econophysics and sociophysics) within the regular (departmental) activities of physics researches! Happily, several main-stream physics journals (like European Physical Journal B, Europhysics Letters, Physica A, Physica D, Physics Letters A, Physical Review E, Physical Review Letters, Journal of Physica A: Mathematical and General, Journal of Physics: Condensed Matter, International J of Modern Physics B, International J of Modern Physics C, and review journals like Physics Reports, Reports on Progress in Physics etc) are regularly publishing research papers in econophysics and sociophysics for the last six-seven years. Like in the initial stages of astrophysics and biophysics, there are some similarities in criticisms from the mainstream economists who essentially tend to ignore these developments in view of their fixed mind-set (of axiomatic foundations most often, and occasionally of 'understanding' the 'natural economic and financial phenomena' claimed by each in terms of their own, but mutually orthogonal, ideas). We believe, however, there are signs of mutual reconciliations emerging. In particular, the balanced emphasis on observations and on developing rigorous analysis of 'toy' models for comprehending only one or some crucial feature(s) of such observations (and not, to start-with, attempt for all the known aspects of the observations), a culture mainly contributed by the econophysicists recently, will help both the streams, physics and economics, in healthy developments.

In short, we believe, criticisms for any such new development are only too natural and have not been uncommon earlier. (Before early sixties, astrophysics was not considered worthy of Nobel prize or for that matter, even for regular funding. Pioneering astrophysicists like M. N. Saha (1893-1956), had to undertake projects in 'mainstream' physics of nuclear science in those days to continue their researches!). Anyway, the previous 'successes' with astrophysics (interdisciplinary science of physics and astronomy), biophysics (interdisciplinary science of physics and biology) and the 'not-so-impressive successes' of geophysics (interdisciplinary science of physics and geology) can indeed help us showing the way to succeed with econophysics/sociophysics too. We all should remember, the criticisms die away because the 'old guards' themselves die away and younger researchers come forward with fresh minds! Also, compare the timescales involved in gaining recognition and successes in both astrophysics and biophysics (or for that matter in geophysics)! With the success already in the last ten or fifteen years in starting the econophysics researches in the physics departments of various universities, of having already a set of very knowledgeable researchers and referees in various established physics journals in appreciating good researches in this interdisciplinary field (and also criticizing the others), we believe, econophysics and sociophysics researches have already scored a critical mass and are poised to make major contributions in science soon (see Fig. 1).

The last 15 years have also seen the popularity of some key books in the field. The book by Rosario Mantegna and H. Eugene Stanley[9] was the first book on Econophysics discussing the key issues related to stock market and finance in general. Betrand Roehner[14] discussed speculations while Jean-Philippe Bouchaud and Marc Potters identified the areas where tools of statistical physics can be used to formulate financial risk and derivative pricing[15]. There were of course many more books that followed which enriched the literature and helped in better understanding of the subject.

Econophysics in Kolkata

Following the early studies of the Travelling Salesman and other multivariate optimization problems, employing classical statistical[16] and quantum mechanical[17] tricks, during 1985-1990, the Kolkata group made some of the earliest modeling investigations regarding the nature of wealth and income distribution in societies and its comparison with the energy distribution in some (quantum) gases. In the 1994 Kolkata Conference, many Indian economists (mainly from Indian Statistical Institute campuses) and physicists discussed about the possible formulations of some of the economic problems and their solutions using tricks from physics[18]. In fact, in one of these papers in the proceedings, possibly the first published joint paper involving both physicist and economist (Sugata Marjit) Indian co-authors[19], the possibility of ideal-gas like model of trading market was discussed. Among other things, it tried to identify, from the known effects of various fiscal policies, the equivalence of the kinetic energy of the gas molecules with the money of the agents in the market and of temperature with the average money in the market. Such a 'finite temperature' gas model of the market was first noted by Dietrich Stauffer (Cologne)[20]. With the possibility of putting more than one agent in the same microstate, identified by the price or money income of the agent in the market, the likely distribution was concluded there[19] to be Bose-Einstein like, rather than Gibbs like. This study of course had the limitation of absence of any comparison with real income distributions in any market or country. In 1995, in the second 'Statphys-Kolkata' series of Conferences (being held in Kolkata for the last one and a half decade now[21]), Gene Stanley (Boston) first introduced the term 'Econophysics' to describe such researches[22]. Since then, Kolkata (erstwhile Calcutta) is considered to be the formal birthplace of this new term: "The term econophysics was ... first used in 1995 at an international conference ... in Calcutta", as mentioned in the successive Symposium homepages of the Nikkei Econophysics Symposia, and also elsewhere (see ref[1]) .

The general features of the observed income/wealth distributions in any society, namely the initial rise of the distribution and then exponential decay (or a log-normal decay) for the majority middle income region (apart from the final Pareto tail for the rich), was taken as an indication that a simple Markov scattering, as in kinetic theory of gas, is insufficient to capture the full trading picture. It was immediately clear that a saving propensity (fraction) for each agent would give the desired feature of a dip at the low income: an agent with the same initial money cannot now become pauper in one scattering or trading as a finite fraction will be saved and can become so only if he/she loses in every successive trading. This study was first done with Anirban Chakraborti[23]. Actually, a little before its publication, Victor Yakovenko and his collaborators (Maryland)[24] had put their seminal paper on the ideal (classical) gas model of income distribution in the cond-mat (electronic) archive and later published (also giving the US data to support their ideal gas model). In fact, a formal

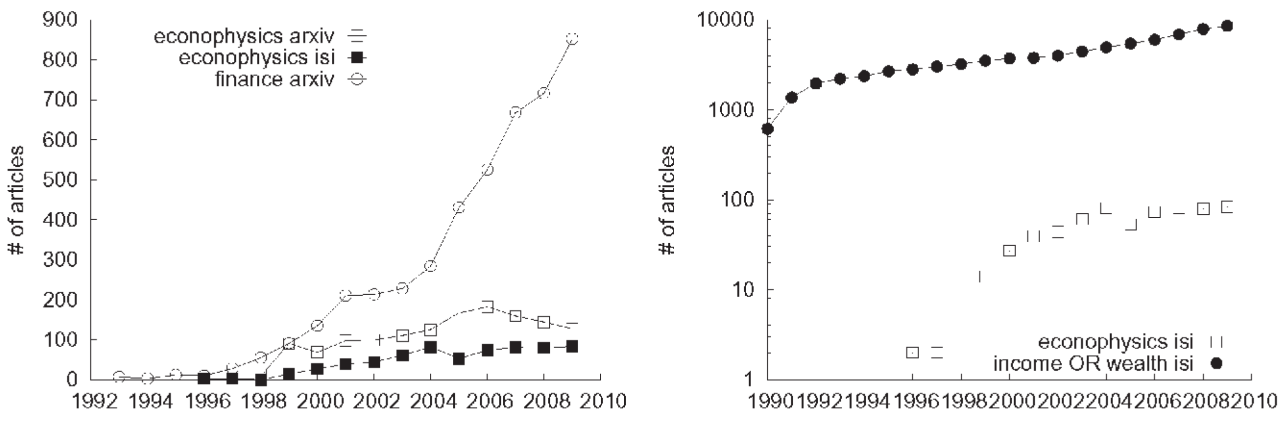

Fig. 1. Growth of econophysics and economics/finance paper is ISI Web of Science journals and Arxiv.

kinetic exchange model was already being developed by Redner et al[25] (Boston University) since 1998. However, the savings model turned out to be very similar to a model proposed by John Angle in mid 80's[26]. Nevertheless, this observation stimulated the Kolkata group very much and noting the advantage of the saving factor in explaining the initial dip in the distribution, over the Gibbs distribution in the ideal gas model of market, several extensions were made: Srutarshi Pradhan and coworkers analyzed the self-organizing property of such models[27], Sitabhra Sinha (Chennai) made a detailed investigation[28] on the stochastic map equivalents of such models, and Anirban Chakraborti, together with Marco Patriarca (Helsinki) and Kimmo Kaski (Helsinki) made an extensive numerical study of the ideal ideal gas model with fixed savings and proposed[29] the Gamma distribution for the steady state income distribution in the model. However, a simple observation by Arnab Chatterjee in late 2002, introducing randomly distributed saving propensity in the same ideal gas model, proved very successful in capturing all the important features of the observed income/wealth distributions: dip for low income group, exponential (Gibbs) decay for the middle income group and power-law (Pareto) tail for the rich people! This was first reported[30] in the 2nd Nikkei Econophysics Symposium in Tokyo in November 2002.

In the meantime, there were several regular and 'popular science' articles which tried to explain and also justify the use of stochasticity in such gas models of markets: for example, Brian Hayes (American Scientist) argued how a little mismatch over the 'just price' of any commodity, as induced by common bargain capacity of the agents in the market, eventually leads to a stochastic gas model "I had accidentally created", which he discovered "to be the same as the" Kolkata model[31]. This kind of spontaneous rediscovery of the gas model for the market independently by several groups indicate perhaps the inevitability of the model.

The fixed saving propensity gas model was later analyzed and improved by several groups. Arnab Das and Sudhakar Yarlagadda here wrote a Boltzmann-like equation for the income probability density, which they solved numerically for the steady state[32]. With Subhrangshu Manna, extensive numerical studies were made on the distributed savings model[33, 34] and the Pareto behavior of the large income tail was established. Infact, together with Debashish Chowdhury (Kanpur), Kimmo Kaski (Helsinki) and Janos Kertész (Budapest), a Conference on "Unconventional Applications of Statistical Physics" was held in Kolkata in early 2003[35], where several groups made further numerical and analytical studies on the Kolkata models, and established several robust features. The data for the Indian income distribution has also been analyzed recently by Sitabhra Sinha[36]. Robin Stinchcombe (Oxford) collaborated with us in solving analytically the master equation for the random saving gas model of the market[37]. A few new results were subsequently found[38–40] and written up. Peter Richmond (Dublin) and his group tested the moments related to the Gamma distribution conjecture for the fixed savings model[41]. Our models and related ones have drawn attention of Giuseppe Toscani and his collaborators and recently they have been instrumental in analyzing the structure of these models[42].

Pradeep Mohanty[43] proposed a treatment of the average wealth of individuals, thus formulating a simple solution of the problem for the case of distributed savings. Abhijit Kargupta[43–46] analyzed the exchange matrix in the general framework as also in the specific cases, and provided important insight into the necessary and sufficient conditions for obtaining the observed distributions. He also provided a simple way to look into Mohanty's solution[47]. Urna Basu and Pradeep Mohanty[48] argued that previously observed distributions from models under the microcanonical framework can be relaxed under certain circumstances, allowing non-conservation of wealth but conserving the mean of the fluctuation, and can produce the same kind of distributions. Recently, a different treatment has been given to the above models under a microeconomic framework[49–52].

Jenny Hogan (New Scientist) in her recent report[53] on these developments described briefly the Kolkata models and mentioned that this "more sophisticated model" (with saving factor) has some added desirable features over the ideal gas model of markets. She additionally reported some interesting (and a few inspiring) opinions of several distinguished economists and physicists on these developments. She also described the 2005 Kolkata workshop on Econophysics as "the first ever conference on Econophysics of Wealth Distributions" where "economists will join physicists to discuss these issues". We indeed believe that some of these wishful successes and developments have already started taking place!

In the context of resource utilisations resulting from repetative games of many agents, evolving their game strategies in parallel, the "Kolkata Paise Restaurant problems"[54–56] were introduced, and a brief demonstration on this problem is now available in the Wolfram Demonstration site[57].

The editorial of *Topical Issue on Physics in Society*[58] mentions two of our publications in an *Editorial Choice-list* of 21 *exemplifying pioneering* publications (earliest in 1872) in Economy & Political Economy. Markowich's book[59] discusses the pioneering papers from Chakrabarti's research group. The term Econophysics has now entered as an entry in the The New Palgrave Dictionary of Economics[1], which indicates that it is now considered as a discipline in economics. Encyclopedia of Complexity & System Science also registered the influential papers from the Kolkata school[60]. The recent economic crisis has triggered a lot of debate[61] regarding the key issues, causes and possible solutions, and mentions the work of the "Kolkata school" in a recent commentary. The specific problem of wealth distribution has also been discussed[62]. Perhaps the most important of all such developments in this context is the publication of the first review article in Reviews of Modern Physics on an important topic in econophysics of wealth distributions[2], discussing also about influential and elegant papers from the "Kolkata School".

Recent times have also seen a lot of heated debate regarding the focus of econophysics research and its relevance to recent economic crisis. The failure of traditional economic theories to predict and handle the worldwide financial crisis has sparked debate among economists as well as scientists working in this interdisciplinary area. Some economicsts have expressed their concern in the approach traditionally taken by physicists to address problems from economics, citing them as Worrying trends in econophysics[13], which have also been addressed appropriately from physicists[63,64].

Econophysics in Kolkata, has seen a lot of activity in recent years. It was in 2005, that we initiated the ECONOPHYS-KOLKATA series of workshops. The goal was to bring together physicists, economists, mathematicians, financial analysts, sociologists from all over India and across the world, well known in their respective fields. These workshops have been highly eventful, apart from the fact that there has been a fruitful exchange of ideas, certain issues regarding the future and scope of Econophysics have been thoroughly discussed in the panel sessions. The proceedings of these workshops[65–69] have registered all the related articles and key discussions[3–5] which have turned out to be important in course of time. Encouraged by the activity, we also edited a book on Econophysics and Sociophysics[70] to collect a few articles of recent interest in the closely related areas, and this book turned out to be very popular. Recently, with Sitabhra Sinha and Anirban Chakraborti, we have written a textbook on Econophysics[71]. ❐

References

1. The New Palgrave Dictionary of Economics, 2nd Ed., Vol 2, p 729, Macmillan, NY (2008).
2. V. M. Yakovenko, J. Barkley Rosser, *Jr. Rev. Mod. Phys.* **81** 1703 (2009).
3. B. K. Chakrabarti, in Econophysics of Stock and other Markets, A. Chatterjee, B. K. Chakrabarti (Eds.), Springer, Milan (2006) p 219.
4. B. K. Chakrabarti, in Econophysics of Markets and Business Networks, A. Chatterjee and B. K. Chakrabarti (Eds.), Springer, Milan (2007) p 251.
5. B. K. Chakrabarti, in Econophysics of Wealth Distributions, A. Chatterjee, S. Yarlagadda, B. K. Chakrabarti (Eds.), Springer, Milan (2005) p 225.
6. http://www.unifr.ch/econophysics/blogs/?p=10; http://www.unifr.ch/econophysics/editorial/show/id/51
7. http://ineteconomics.org
8. B. B. Mandelbrot, *J. Business* **36** 394 (1963).
9. An Introduction to Econophysics: Correlations and Complexity in Finance, R. N. Mantegna and H. E. Stanley, Cambridge University Press, Cambridge (1999).
10. Minority Games, D. Challet, M. Marsili and Y-C. Zhang, Oxford University Press, Oxford, (2005).
11. T. Lux and M. Marchesi, *Nature* **397** 498 (1999).
12. A. Chakraborti, I. Muni Toke, M. Patriarca and F. Abergel, arXiv:0909. (1974).
13. M. Gallegati, S. Keen, T. Lux and P. Ormerod, *Physica* **A 370** 1 (2006).
14. Patterns of Speculation - A Study in Observational Econophysics, B. Roehner, Cambridge University Press, Cambridge (2002).
15. Theory of Financial Risk and Derivative Pricing, J-P. Bouchaud, M. Potters, Cambridge University Press, Cambridge (2003).
16. See, e.g., P. Bhattacharyya and A. Chatterjee, in Statistics of Linear Polymers on Disordered Media, B.K. Chakrabarti (Ed.), Elsevier, Amsterdam (2005) p 271.
17. Quantum Annealing and Related Optimisation Methods, A. Das and B. K. Chakrabarti (Eds.) LNP **vol. 679**, Springer-Verlag, Heidelberg (2005); A. Das and B. K. Chakrabarti, *Rev. Mod. Phys.* **80** 1061 (2008).
18. Proc. School on Complex Systems, Kolkata *Indian J. Phys.* **B 69** 483 (1995).
19. B. K. Chakrabarti and S. Marjit, *Ind. J. Phys.* **B 69** 681 (1995).
20. See, e.g., S. M. de Oliveira, P. M. C. de Oliveira and D. Stauffer, Evolution, Money, War and Computers, Tuebner, Stuttgart (1999) p 127.
21. Statphys-Kolkata I-V: *Procs. in Physica* **A 186** (1 & 2) (1992) I; *Physica* **A 224** (1 & 2) (1996) II; *Physica* **A 270** (1 & 2) (1999) III; *Physica* **A 318** (1 & 2) (2003) IV; *Physica* **A 346** (1 & 2) (2005) V.
22. H. E. Stanley, *Physica* **A 224** 302 (1996).
23. A. Chakraborti and B. K. Chakrabarti, *Eur. Phys. J.* **B 17** 167 (2000).
24. A. A. Dragulescu, V. M. Yakovenko, *Eur. Phys. J.* **B 17** 723 (2000).
25. S. Ispolatov, P. L. Krapivsky and S. Redner, *Eur. Phys. Jour.* **B 2** 267 (1998).
26. J. Angle, *Social Forces* **65** 293 (1986); *Physica* **A 367** 388 (2006).
27. A. Chakraborti, S. Pradhan and B. K. Chakrabarti, *Physica* **A 297** 253 (2001).
28. S. Sinha, *Phys. Scripta* **T 106** 59 (2003).
29. M. Patriarca, A. Chakraborti and K. Kaski, *Phys. Rev.* **E 70** 016104 (2004).
30. B. K. Chakrabarti and A. Chatterjee, in Applications of Econophysics (Proc. Second Nikkei Symposium on Econophysics, Tokyo, 2002), Ed. H. Takayasu, Springer-Verlag, Tokyo pp. 280-285 (2004).
31. B. Hayes, *American Scientist* **90** (Sept-Oct) 400 (2002).
32. A. Das and S. Yarlagadda, *Phys. Scripta* **T 106** 39 (2003).
33. A. Chatterjee, B. K. Chakrabarti and S. S. Manna, *Phys. Scripta* **T 106** 36 (2003).

34. A. Chatterjee, B. K. Chakrabarti and S. S. Manna, *Physica* **A 335** 155 (2004).
35. Physics of Random Networks, Econophysics and Models of Biophysics and Sociophysics, *Phys. Scripta* **T 106** 1 (2003).
36. S. Sinha, *Physica A* **359** 555 (2006).
37. A. Chatterjee, B. K. Chakrabarti and R. B. Stinchcombe, *Phys. Rev.* **E 72** 026126 (2005).
38. A. Chatterjee and B. K. Chakrabarti, *Eur. Phys. J.* **B 54** 399 (2006).
39. A. Chatterjee and B. K. Chakrabarti, *Eur. Phys. J.* **B 60** 135 (2007).
40. A. Chatterjee, S. Sinha and B. K. Chakrabarti, *Current Science* **92** 1383 (2007).
41. P. Repetowicz, S. Hutzler and P. Richmond, *Physica A* **356** 641 (2005).
42. B. Düring and G. Toscani, *Physica A* **384** 493 (2007); B. Düring, D. Matthes, G. Toscani, *Phys. Rev.* **E 78** 056103 (2008); D. Matthes, G. Toscani, *J. Stat. Phys.* **130** 1087 (2008); D. Matthes, G. Toscani, *Kinetic and related Models* **1** 1 (2008); V. Comincioli, L. Della Croce, G. Toscani, *Kinetic and related Models* **2** 135 (2009).
43. P. K. Mohanty, *Phys. Rev.* E **74**) 011117 (2006.
44. A. Kargupta, *Physica A* **359** 634 (2006).
45. A. Kargupta, *Comm. Comp. Phys.* **1** 503 (2006).
46. A. Kargupta, *Physica A* **387** 6819 (2008).
47. A. Kargupta, in [70] pp. 161.
48. U. Basu and P. K. Mohanty, *Eur. Phys. J.* **B 65** 585 (2008).
49. A. S. Chakrabarti and B. K. Chakrabarti, *Physica* **A 388** 4151 (2009).
50. A. S. Chakrabarti and B. K. Chakrabarti, *Economics E-journal*, 4 (2010): http://www.economics-ejournal.org/economics/journalarticles/2010-4
51. A. S. Chakrabarti and B. K. Chakrabarti, *Physica* **A 389** 3572 (2010).
52. A. S. Chakrabarti, arXiv:1006.5044.
53. J. Hogan, There is one Rule for the Rich, New Scientist, UK, 12 March 2005, pp 6-7.
54. A. S. Chakrabarti, B. K. Chakrabarti, A. Chatterjee and M. Mitra, *Physica* **A 388** 2420 (2009).
55. A. Ghosh, A. S. Chakrabarti and B. K. Chakrabarti, in [68] pp 3-9.
56. A. Ghosh, A. Chatterjee, M. Mitra, B. K. Chakrabarti, *New J. Phys.* **12** 075033 (2010).
57. A. Ghosh, B. K. Chakrabarti, (2009), Kolkata Paise Restaurant (KPR) problem, http://demonstrations.wolfram.com/KolkataPaiseRestaurantKPRProblem.
58. *Eur. Phys. J.* **B 57** 121 (2007).
59. Applied Partial Differential Equations, P. A. Markowich, (Springer, Berlin) (2007).
60. Encyclopedia of Complexity & System Science, **Vol. 3**, pp 2800, (Springer, New York) (2009).
61. *Nature Physics* **5** 2 (2009).
62. World's economies show similarities in economic inequality, PhysOrg.com, April 6, 2007: http://www.physorg.com/news95074548.html
63. P. Richmond, B. K. Chakrabarti and A. Chatterjee, *J. Angle*, in [66], pp. 244 (2006).
64. J. L. McCauley, arXiv:physics/0606002.
65. Econophysics of Wealth Distributions, A. Chatterjee, S. Yarlagadda and B. K. Chakrabarti (Eds.), (Springer, Milan) (2005).
66. Econophysics of Stock and other Markets, A. Chatterjee, B. K. Chakrabarti (Eds.), (Springer, Milan) (2006).
67. Econophysics of Markets and Business Networks, A. Chatterjee, B. K. Chakrabarti (Eds.), (Springer, Milan) (2007).
68. Econophysics & Economics of Games, Social Choices and Quantitative Techniques, B. Basu, B. K. Chakrabarti, S. R. Chakravarty, K. Gangopadhyay (Eds.), (Springer, Milan) (2009).
69. Econophysics of Order-driven Markets, F. Abergel, B. K Chakrabarti, A. Chakraborti, M. Mitra (Eds.), Springer, Milan (2010, to be published).
70. Econophysics and Sociophysics, B. K. Chakrabarti, A. Chakraborti, A. Chatterjee (Eds.), Wiley-VCH, Berlin (2006).
71. Econophysics: An Introduction, S. Sinha, A. Chatterjee, A. Chakraborti, B. K. Chakrabarti, Wiley-VCH, Berlin (2010), in press).